Dynamics of an active magnetic particle in a rotating magnetic field

A. Cēbers*
Institute of Physics, University of Latvia, Salaspils-1, LV-2169, Latvia

M. Ozols
University of Latvia, Zellu-8, Riga, Latvia

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The motion of an active (self-propelling) particle with a permanent magnetic moment under the action of a rotating magnetic field is considered. We show that below a critical frequency of the external field the trajectory of a particle is a circle. For frequencies slightly above the critical point the particle moves on an approximately circular trajectory and from time to time jumps to another region of space. Symmetry of the particle trajectory depends on the commensurability of the field period and the period of the orientational motion of the particle. We also show how our results can be used to study the properties of naturally occurring active magnetic particles, so-called magnetotactic bacteria.

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Active systems interacting with electromagnetic fields are interesting from different points of view [1,2]. The pumping of liquid without force due to the negative viscosity effect of dielectric suspensions [3], the flexible magnetic filaments interacting with an ac magnetic field [4,5], and their self-propulsion in liquid [6,7] may also be mentioned here among other examples. Active magnetic systems also exist in nature—the magnetotactic bacteria have a biochemical machinery allowing them to produce ferromagnetic particles inside their bodies and use the particles to orientate in the magnetic field of the Earth [8,9].

Active magnetic systems have very interesting properties which have not been completely investigated yet. In this paper we consider an active particle with a permanent magnetic dipole under the action of a rotating magnetic field. The study of the behavior of magnetic particles in a rotating magnetic field has a rather long history (see, for example, Ref. [10] for further references). Among the most recent developments in this field we can mention Ref. [11], where the motion of an anisotropic particle in the rotating optical field is investigated. Several regimes of the particle motion in a rotating magnetic field are established—synchronous motion at frequencies below the critical and back and forth motion at frequencies above it [12]. Interesting features occur in the behavior of flexible magnetic particles under the action of the rotating field [13,14]. In spite of this long-standing interest in the behavior of different particles in ac magnetic fields, active particles (self-propelled in a liquid) have never been properly investigated. An indication that a lot of interesting things can take place in this case can be found in the paper [15], where from the figures one can see that at the frequencies above the critical frequency the character of magnetotactic bacteria motion changes drastically. Instead of motion along the circles which occurs if the frequency is low enough, the bacteria starts to jump between the circles of a smaller radius at higher frequencies. Such a regime of the particle motion has never been studied before. Here we present a simple model of the behavior of an active particle with a permanent magnetic moment in a rotating magnetic field. It turns out that a simple combination of active properties of a particle with its capability to orientate along the applied field leads to rather rich behavior which has interesting possibilities for different practical applications among which the determination of the physical properties of the magnetotactic bacteria should be mentioned.

Let us introduce our model. The magnetic particle, due to its self-propulsion, moves with velocity v in the direction of its magnetic moment (this mechanism is used by magnetotactic bacteria for their survival in the environment [8,9,16]). Keeping in mind magnetotactic bacteria, as the example, we do not consider the possibility of their tumbling [17]. Although some of them have only one flagella, nevertheless there are exceptions [18]. Influence of tumbling on the motion of magnetotactic bacteria remains an interesting issue to study in the future. Besides this the active particle is approximated by an axisymmetric body. It should be noted that helicoidal trajectories are observed for some magnetotactic bacteria due to their nonaxisymmetry [18]. Its influence on motion of bacteria in the rotating field is an interesting issue to study in the future also. The coupling of the translational and rotational degrees of freedom of the bacteria are essential, for example, in their motion near solid walls [19].

The magnetic moment orientates along the applied field H. The kinetics of orientation is determined by the torque balance acting on the particle,

$$-\alpha\frac{d\vartheta}{dt}+MH\sin\beta=0. \tag{1}$$

Here M is the dipole moment of the particle, α is the rotational friction coefficient, ϑ is the particle orientation angle with respect to some fixed direction, which we take to be the x axis, and β is the angle between the magnetic field and magnetic moment of the particle. Influence of the thermal fluctuations on the orientational dynamics of the particle is

*Electronic address: aceb@tesla.sal.lv

neglected. Keeping in mind the magnetotactic bacteria as active magnetic particles, it is justified by the large values of their Langevin parameter already in the small magnetic fields [16,20].

In the case of a rotating field $\beta=\omega t-\vartheta$ and the equations of the particle motion are as follows:

$$\frac{dx}{dt}=v\cos\vartheta, \tag{2}$$

$$\frac{dy}{dt}=v\sin\vartheta, \tag{3}$$

$$-\alpha\frac{d\vartheta}{dt}+MH\sin(\omega t-\vartheta)=0. \tag{4}$$

Equation (4) can be put in the form

$$\frac{d\beta}{dt}=\omega-\omega_c\sin\beta. \tag{5}$$

Here,

$$\omega_c=MH/\alpha. \tag{6}$$

Equation (5) has a stationary solution $\beta=\beta_0$ at $\omega \leqslant \omega_c$: $\sin\beta_0=\omega/\omega_c$. If $\omega>\omega_c$ then the particle cannot rotate synchronously with the applied field and angle β is a periodic function of time. A simple integration gives ($\gamma=\omega_c/\omega < 1$),

$$\beta=2\arctan[\gamma+\sqrt{1-\gamma^2}\tan\{\sqrt{1-\gamma^2}\omega(t-t_0)/2\}]. \tag{7}$$

Since the time moment t_0 can be chosen arbitrarily, we will choose it to be zero.

Since the speed of the particle is constant its trajectory has simple geometrical properties. Introducing tangent $\vec{t}$ and normal $\vec{n}$ of the trajectory, which are connected by the Frenet formula $d\vec{t}/dl=-k\vec{n}$, where l is the natural parameter of the trajectory (its contour length) but k is its curvature, we have

$$\frac{d(v\vec{t})}{dt}=-v^2k\vec{n} \tag{8}$$

and as a result

$$k=\frac{1}{v}\left(\omega-\frac{d\beta}{dt}\right). \tag{9}$$

In a synchronous regime when $d\beta/dt=0$ we have $k=\omega/v$. It means that the trajectory of a particle is a circle with a radius which diminishes with the frequency of the rotating field. In a nonsynchronous regime we obtain $k=\omega_c\sin\beta/v$. This means that the trajectory consists of stages with alternately changing direction of curvature. If $\beta\in\,](2n)\pi;\ (2n+1)\pi[$ then the curvature is positive ($k>0$), but if $\beta\in\,](2n+1)\pi;\ (2n+2)\pi[$ then the curvature is negative ($k<0$), where $n=0,1,\ldots$. Switching from one type of motion to another takes place at time moments when $\sin\beta=0$. If $\gamma<1$ is close to 1 the trajectory of a particle looks like motion along many circles with fast switching between them as may be seen from the plot of the dimensionless curvature $\sin\beta$ (as it depends on time), which is shown in Fig. 1. There are photos in this paper [15] that can serve as evidence that this type of active particle motion indeed takes place. Thus for a long period of time the trajectory of the particle has an almost constant curvature and only during fast jumps from one circle to another does the curvature change significantly and becomes negative.

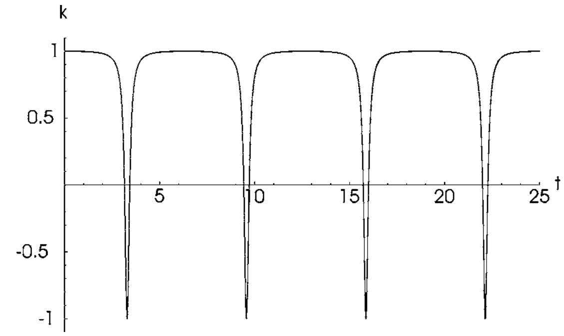

FIG. 1. Curvature of trajectory in dependence on time. $\gamma=0.99$.

It is possible to find some characteristics of this motion. A winding number Wn gives the change of the particle phase ϑ per period of its orientational motion. It reads

$$\mathrm{Wn}=\frac{\Delta\vartheta}{2\pi}=\frac{\gamma}{2\pi}\int_0^{2\pi}\frac{\sin\beta d\beta}{1-\gamma\sin\beta}. \tag{10}$$

A simple integration gives

$$\mathrm{Wn}=\frac{1}{\sqrt{1-\gamma^2}}-1. \tag{11}$$

Dependence of Wn on γ is shown in Fig. 2. We see that a particle at the rotating field frequency close to the critical frequency makes a lot of turns before switching to the next circular trajectory. For γ values less than $\sqrt{3}/2\simeq 0.866$ the Wn number is less than 1 and the particle does not make a full turn before switching to another part of space. Characteristic trajectories of the particle motion are illustrated in Fig. 3 for several values of the parameter γ: $\sqrt{1-\gamma^2}=p/q$ ($q=25$; $p=2$, 3, 4, 6, and 7). We see that the numerator in the ratio p/q, if p and q do not have a common divider, defines the number of the parts of space which the particle visits

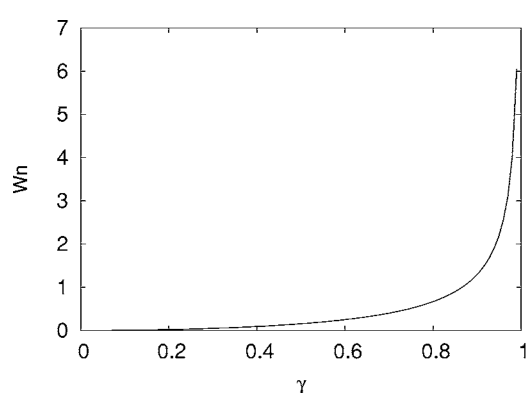

FIG. 2. Winding number in dependence on $\gamma=\omega_c/\omega$.

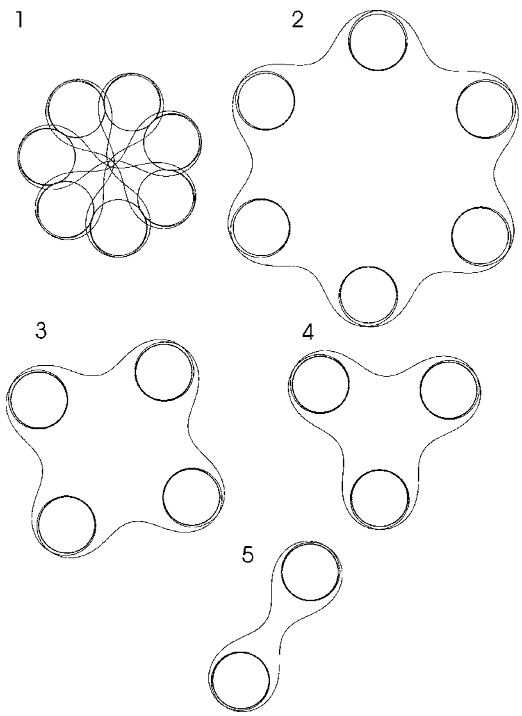

FIG. 3. Trajectories of the particle for $\sqrt{1-\gamma^2}=7/25$ (1); $6/25$ (2); $4/25$ (3); $3/25$ (4); $2/25$ (5).

during its periodical regime of motion. In the case when the numerator is equal to 1 the trajectory of the particle on average is a straight line [Fig. 4 ($q=25$; $p=1,5$)] and the winding number Wn is equal to $q-1$, as it should be.

It is easy to see that the type of particle motion depends on the commensurability of the field period $2\pi/\omega$ and the period of the orientational motion $T_1=(2\pi/\omega)/\sqrt{1-\gamma^2}$. This gives the condition of the periodicity: $\sqrt{1-\gamma^2}=p/q$ is rational. Examples of periodical trajectories which correspond to $q=25$ and different p are shown in Fig. 3. In the case when the periods are not commensurable, the trajectory covers the accessible space region densely. Two close values of the parameter $\sqrt{1-\gamma^2}$ ($p/q=\sqrt{7}/4$ and $p/q=2/3$) are illustrated in Fig. 5.

The distance L_f between the end points of the part of trajectory with $k<0$, which characterizes the distance between the regions of space explored by a particle, can be calculated by numerically integrating the equations of motion (2) and (3). L_f in the units of v/ω_c—the radius of a circle at a critical frequency—is shown in Fig. 6. We see that

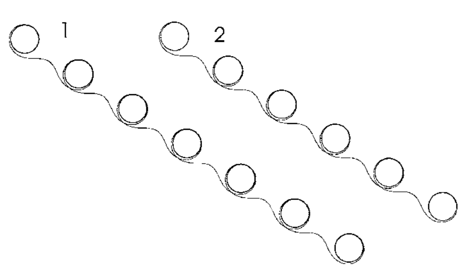

FIG. 4. Trajectories of the particle for $\sqrt{1-\gamma^2}=5/25$ (1); $1/25$ (2).

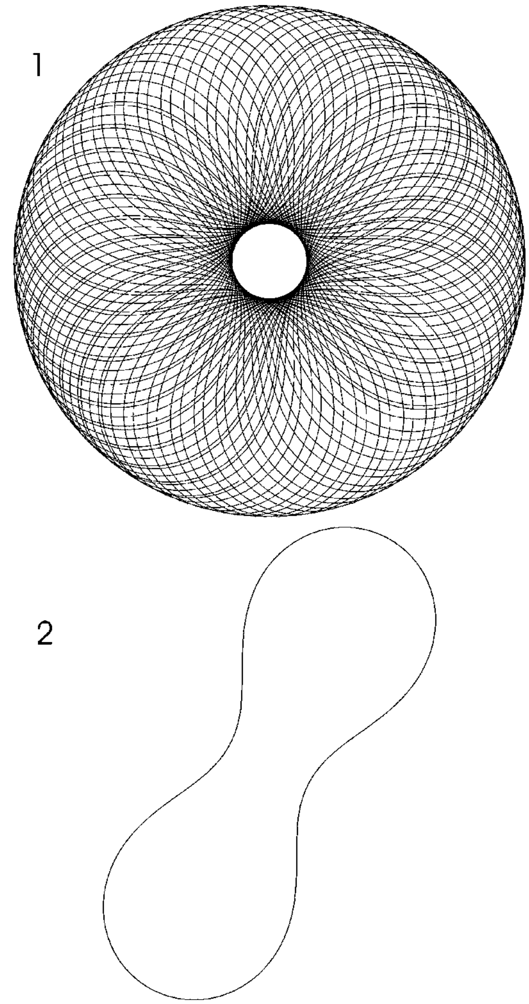

FIG. 5. Trajectories of the particle for an incommensurable [$p/q=\sqrt{7}/4$ (1)] and commensurable [$p/q=2/3$ (2)] ratio of frequencies.

at frequencies close to the critical frequency ω_c the distance between two regions of the circular motion of the particle is about the diameter of the circle. Less trivial and more interesting is the dependence of the radius of circle R, which encloses the region of space visited by an active particle, on ω_c/ω. From Fig. 3 and Fig. 4 it is clear that it should be nonmonotonous. $1/R$ in dependence on winding number Wn

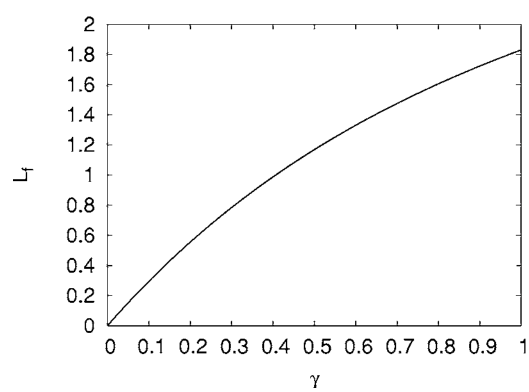

FIG. 6. Length of particle jumps in dependence on $\gamma=\omega_c/\omega$.

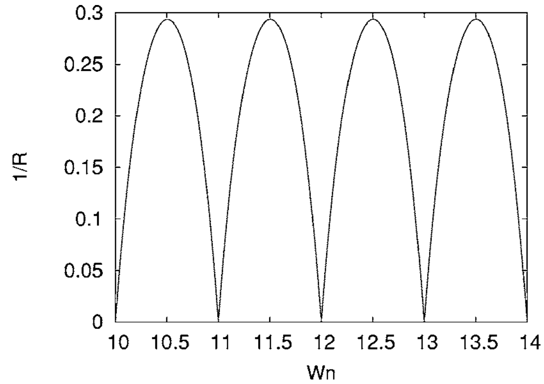

FIG. 7. Curvature of enclosing circle in dependence on winding number Wn.

is shown in Fig. 7. It should be noted that the resulting particle circulation inside the enclosing circle can be both clockwise and counterclockwise in dependence on Wn. The direction of the circulation is undefined if the particle goes through the center of the explored region (see Fig. 8, Wn $=6.5538$). This takes place when the center coincides with the position of the particle at the time moments $(\pi+\arccos\gamma)/\sqrt{1-\gamma^2}+iT_1$ (i is a natural number). The distance Δr between the starting and final points of the particle trajectory for the period of its orientational motion in dependence on the winding number is shown in Fig. 9. For winding number values close to the critical (when trajectory goes through the center) very fast circulation around the center can be observed. From Fig. 9 one can see that the critical values of the winding number are slightly above $k+1/2$, where k is a natural number. Numerical calculations show that $\mathrm{Wn}-(k+1/2)$ is close to 0.0538 for large k. One can set $\mathrm{Wn}=n/(2n+1)$ which is always less than 1/2 and approaches it as n increases. For such a winding number, circulation will be very fast and the particle will make n (the numerator of Wn) turn around the center of the trajectory until returning to the starting point. When $\mathrm{Wn}=k+n/(2n+1)$, where k is a natural number, the particle makes k small turns in each symmetric piece of trajectory (in addition to the fast circulation).

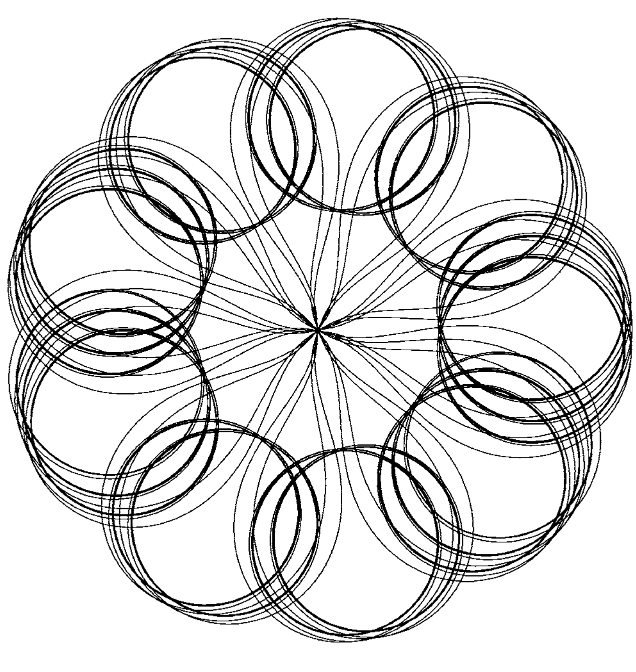

FIG. 8. Trajectory of the particle crossing the center of the explored region. Wn=6.5538.

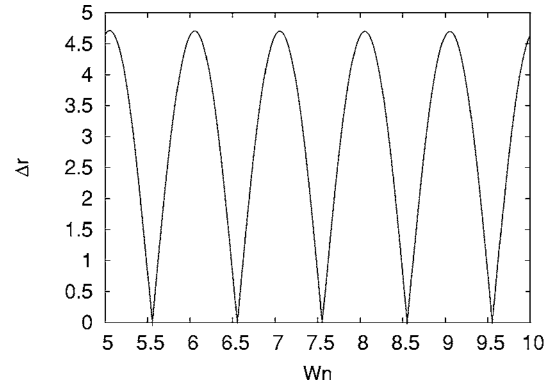

FIG. 9. Distance between the starting and final positions of the particle per period of its orientational motion in dependence on the winding number.

The amount of available experimental data on motion of active particles in the rotating field is very scarce. The only data available to us are given in Ref. [15] where the photos of the magnetotactic bacteria trajectory for the two frequencies of the rotating field 0.1 Hz and 0.4 Hz are shown. In the first case the trajectory of the bacteria is a circle, while in the second case the jumps between the circles of the smaller radius can be seen. From the radius of circle $R_{0.1}\simeq 29.7\ \mu\mathrm{m}$ and the rotating field frequency, the velocity of the bacteria according to the relation (9) can be determined $v=\omega R_{0.1}\simeq 18.7\ \mu\mathrm{m/s}$. This is a reasonable value for the swimming velocity of the magnetotactic bacteria [16]. A ratio of radii of the circles at 0.1 Hz and 0.4 Hz ($R_{0.4}\simeq 9.4\ \mu\mathrm{m}$) allows one to determine the critical angular frequency of the rotating field: $\omega_c=\omega R_{0.1}/R_{0.4}\simeq 2$ Hz. Taking for the magnetic moment of the magnetotactic bacteria M the reasonable value $2\times10^{-12}\ \mathrm{G\,cm^3}$ [8,16] (let us note that the magnetic moment of bacteria shown in Fig. 2 of [15] is 10–50 times less than the magnetic moment of magnetobacterium bavaricum containing more than 500 magnetosomes, which were studied by a TEM microscopy in Ref. [15]) the relation for the critical frequency (6) at the known value of the magnetic field $H=1.6$ Oe allows us to determine the rotational drag coefficient α of bacteria 1.6×10^{-12} erg s. Using the dependence of the rotational drag coefficient on the long a and short b axes of the ellipsoidal body with volume V in the liquid with viscosity η (we take $\eta=1$ cP for an estimate),

$$\alpha = 8\pi\eta V\frac{a^2+b^2}{a^2 n_1 + b^2 n_2}, \qquad (12)$$

where n_1 and n_2 are the depolarization coefficients of ellipsoid with eccentricity e,

$$n_1 = \frac{2\pi(1-e^2)}{e^3}\left(\ln\frac{1+e}{1-e} - 2e\right) \tag{13}$$

and

$$n_2 = \frac{1}{2}(4\pi - n_1). \tag{14}$$

The value of the critical frequency gives a reasonable value of the length of the bacteria with $b=0.25$ μm: $2a=7.7$ μm. The last estimate of the parameters of the effective ellipsoid is rather illustrative since the shapes of magnetotactic bacteria are not exact ellipsoids and an important contribution to its rotational drag coefficient also comes from the attached flagellas. Since the magnetic moments of bacteria can be determined by magnetic measurements the investigation of their motion in the rotating field can be used for the determination of the rotational drag coefficient, which is a rather difficult problem.

In this paper we have shown that a simple model of an active magnetic particle under the action of a rotating magnetic field can imitate the experimentally observed trajectories of magnetotactic bacteria. Since it is possible to make such particles artificially [4,6,7], rather interesting possibilities of their application for the enhancement of mass transfer arise. This problem has already attracted the attention of other researchers [21].

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